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CONFIDENCE LIMITS FOR THE RATIO OF TWO MEANS

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The method to determine confidence limits for the ratio of the means ξ and η of two variates with a two-dimensional normal distribution, usual in biological assays (E. C. Fieller, 1944), can be brought into the following more general form.

General formulation.

To determine confidence limits for $\alpha = \frac{\xi}{\eta}$ we try to find five functions $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{a}_{11}, \boldsymbol{a}_{12}$ and $\boldsymbol{a}_{22}$ of the observations $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_k$, such that:

1. $\boldsymbol{x}$ and $\boldsymbol{y}$ are $N(\xi, \eta; \sigma_{11}, \sigma_{12}, \sigma_{22})$ distributed with unknown parameters;

2. $$\boldsymbol{z} \overset{\text{def}}{=} \eta \boldsymbol{x} - \xi \boldsymbol{y} \tag{1}$$

and

$$\boldsymbol{s_z} \overset{\text{def}}{=} + \sqrt{\eta^2 \boldsymbol{a}_{11} - 2\eta\xi \boldsymbol{a}_{12} + \xi^2 \boldsymbol{a}_{22}} \tag{2}$$

are independently distributed;

3. for some known integer f

$$\frac{f \boldsymbol{s}_{\boldsymbol{z}}^2}{\sigma_{\boldsymbol{z}}^2}, \text{ with } \sigma_{\boldsymbol{z}}^2 = \eta^2\sigma_{11} - 2\eta\xi\sigma_{12} + \xi^2\sigma_{22}$$

has a χ_f^2-distribution.

From these conditions it follows that $\boldsymbol{t} = \frac{\boldsymbol{z}}{\boldsymbol{s_z}}$ has Student's-distribution with f degrees of freedom. If t_ε is determined by

$$\mathrm{P}\left[\left|\frac{\boldsymbol{z}}{\boldsymbol{s_z}}\right| \leqq t_\varepsilon\right] = 1 - \varepsilon,$$

then it follows from (1) and (2), that the inequality

$$(\boldsymbol{x}^2 - t_\varepsilon^2 \boldsymbol{a}_{11}) - 2\frac{\xi}{\eta}(\boldsymbol{xy} - t_\varepsilon^2 \boldsymbol{a}_{12}) + \frac{\xi^2}{\eta^2}(\boldsymbol{y}^2 - t_\varepsilon^2 \boldsymbol{a}_{22}) \leqq 0 \tag{3}$$

has a probability $1 - \varepsilon$.

All values α' which, substituted for $\frac{\xi}{\eta}$, satisfy (3) (the value ∞ included), form a confidence interval for $\alpha = \frac{\xi}{\eta}$ corresponding to the confidence level $1 - \varepsilon$.

Examples.

The method may e.g. be applied to the following situations:

a. independent pairs of observations $(\boldsymbol{x}_1, \boldsymbol{y}_1), \ldots (\boldsymbol{x}_n, \boldsymbol{y}_n)$, of $\boldsymbol{x}$ and $\boldsymbol{y}$ are given, where $\boldsymbol{x}$ and $\boldsymbol{y}$ have a simultaneous normal distribution with means ξ and η;

b. independent observations $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n$ and $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_m$ of $\boldsymbol{x}$ and $\boldsymbol{y}$ are given, where $\boldsymbol{x}$ and $\boldsymbol{y}$ are independently normally distributed with mean ξ resp. η and variance $\sigma_{\mathbf{x}}^2$ resp. $\sigma_{\mathbf{y}}^2 = k\sigma_{\mathbf{x}}^2$, provided k is a known constant (biological assays).

The more general case, when the ratio of $\sigma_{\mathbf{x}}^2$ to $\sigma_{\mathbf{y}}^2$ is unknown, cannot be solved by this method.

c. The confidence limits for the slope of a line when both variates are subject to errors (with a twodimensional normal distribution), given by A. Wald (1940), is another example of this method.

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